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THE EFFECT OF STATISTICAL VELOCITY VARIATION ON THE GAUSSIAN BIVARIATE PROBABILITY OF HIT FOR SMALL CALIBER SYSTEMS

May 1975

Munitions Development & Engineering Directorate

U.S. ARMY ARMAMENT COMMAND
FRANKFORD ARSENAL
PHILADELPHIA, PENNSYLVANIA 19137

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The statistical variations in both the average velocity and the linear standard deviation of velocity, due to the location of propellant air-space, are investigated by trajectory conversions to errors on a vertical target and by calculations of rectangular hit probabilities with the Gaussian bivariate distribution. The two cartridges examined are the 7.62mm, Ball, M80 and the 5.56mm, Ball, M193. Sources of errors and their magnitudes are discussed. Two diverse levels of aiming error are over

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20. ABSTRACT (continued)

assumed, corresponding to present requirements for the Future Rifle System. The range-dependent errors due to velocity variations are treated as perturbations of the ballistic error. The nose-tap (NT) procedure of chambering cartridges is compared to the standard base-tap (BT) procedure by assuming that zero bias is applicable to the BT procedure and that the NT procedure introduces a finite bias. Calculations are conducted with assumptions which tend to maximize the influence of the velocity variation, and the limiting case of zero aiming error is also treated. The changes in hit probability due to the statistical velocity variations corresponding to the BT and NT air space positions are shown to be insignificant for these two standard cartridges.

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INTRODUCTION

The existence of air space in small caliber cartridges has been known to influence the muzzle velocity and peak pressure to an extent dependent upon the location and magnitude of the air space. These types of data are statistical in nature and for standardized small caliber cartridges, show rather small divergences in average muzzle velocity and its linear standard deviation. The view put forth in a proposal[1] that the variation in ballistic performance associated with the location of the air space in standard rifle cartridges is critically important has not been supported by technical arguments. The principal argument opposing this criticality ab initio was that the errors associated with the air space position were quite small compared to those errors originating from all other sources. However, this assessment had been qualitative, and the purpose of the present study is to quantify the argument by ultimately calculating differences, due to air space location, in combat hit probability under two diverse levels of assumed aiming error for the 7.62mm, Ball, M80 cartridge fired from the M14 rifle and for the 5.56mm, Ball, M193 cartridge fired from the M16 rifle. For this purpose, the initial requirements are data on the average velocity $\overline{V}$ and its linear standard deviation $\overline{\sigma}_V$ for both of these cartridges, fired with different air space locations.

Two locations are considered: (1) Topside and behind the projectile, resulting from base-tapping the round before chambering horizontally - the standard method of loading; (2) Topside and in front of the primer, resulting from nose-tapping the round prior to chambering.

Table I presents such representative data on the average velocity at 78 feet and its linear standard deviation for the M80 and M193 with lots produced at Lake City and Twin City Army Ammunition Plants (LC and TC). Each value represents about 1000 firings (20 rounds from each of 50 lots). It is observed that the nose tap procedure produces larger values of $\overline{\sigma}_V$ in every case and larger values of $\overline{V}$, except for the M193, TC case. The change in $\overline{V}$ is less than 1 percent, and the change in $\overline{\sigma}_V$ is less than 40 percent.

SOURCES OF ERROR

To compute the hit probability for a single projectile on a known target, it is necessary to define the total delivery error as a linear standard deviation σ_t to be used in the bivariate Gaussian (normal)

1. Moore, L., Proposal-Engineering Test of Small Caliber Rifle-Ammunition Systems, 17 July, 1974.

TABLE 1.

Average Velocities $\bar{V}$ at 78 Feet and Linear Standard Deviations $\bar{\sigma}_V$ of the 7.62mm, Ball, M80 and the 5.56mm, Ball, M193 Cartridges.

		Base Tap (fps)		Nose Tap (fps)	
Cartridge	Lots	$\bar{V}$	$\bar{\sigma}_V$	$\bar{V}$	$\bar{\sigma}_V$
M80	LC*	2745	18	2767	23
M80	TC**	2748	16	2773	21
M193	LC	3246	18	3268	25
M193	TC	3250	21	3244	23

* Lake City Ammunition Lots
** Twin City Ammunition Lots

probability distribution function. For a linear combination of independent random variables, the total variance is

$$\sigma_t^2 = \sigma_1^2 + \sigma_2^2 + \ldots$$

where σ_1^2, σ_2^2, ... are the variances of the independent components. In rifle firings, the two major sources of error are the linear standard deviations due to aiming error σ_A and due to the ballistic or round-to-round error σ_r. In the tactical role of small arms, the combat aiming errors are quite variable, but even at their minimal values are very much larger than ballistic errors.[2 to 6] Therefore, the change in the σ_r and the introduction of a bias, due to the nose-tap procedure, would be expected to have little effect upon combat hit probabilities.

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[2] Carn, R.E., Simmons, R.L., and Sperrazza, J., Comparative Effectiveness Evaluation of the M14 and Other Rifle Concepts (U), Ballistic Research Laboratories, Tech. Note 1482, Dec. 1962. (Secret)

[3] Malinoski, F.A., A Casualty Probability Analysis of Small Arms Weapons Systems of Various Caliber (U), Frankford Arsenal, Report R-1712, March 1964. (Secret)

[4] Malinoski, F.A., and McHugh, R.J., An Effectiveness Analysis of Spin-Stabilized Rifle Systems Based on a Caliber .17 Projectile (U), Frankford Arsenal, Report R-1804, Feb. 1966. (Confidential)

[5] Simmons, R.L., and Carn, R.E., Effectiveness of Small Arms Weapons Systems (U), Ballistic Research Laboratories, Memorandum Report 1764, July 1966. (Confidential)

[6] Carn, R.E., and Fallin, H.K., Effectiveness Comparision of 1:12 and 1:14 Inch Twist Rates for M16A1 Rifle (U), Ballistic Research Laboratories, Memo Report 1886, Dec. 1968. (Secret)

Ballistic errors σ_r of standard small arms cartridges fired from standard weapons are approximately 0.4 mils, a value which remains fairly constant over the tactical range, although an increasing trend occurs at longer ranges, particularly after the traversal of the transonic region. The value of 0.4 mils is supportable by considerable data on the M193 in the M16 rifle and the M80 in the M14 rifle, at least out to 600 meters. The maximum range considered in this study is 500 meters.

It is theoretically possible to further divide the ballistic error σ_r into its components. The variation of muzzle velocity is the source of merely one component of ballistic error. This component, however, has the fortuitous advantage of being easily calculable, in contrast to most of the other sources of σ_r Some of these other highly-interrelated sources are statistical variations in:

1. Projectile design parameters, e.g., mass, center of gravity, moments of inertia, eccentricity.

2. Various aerodynamic coefficients, e.g., drag, moment, yaw-drag, cross-wind, magnus, normal force, damping, spin-deceleration; also stability and yawing motion.

3. Interior ballistic parameters, e.g., muzzle pressure, frictional force and engraving stresses on projectile, gas temperature, heat generated and flux into chamber and barrel, propellant charge, propellant geometry distribution, case volume, primer characteristics.

4. Weapon design characteristics and weapon-ammunition interface parameters, e.g., chamber, barrel, rifling configuration, thermal expansions, characteristics of gas flow for cycling, erosion, clearance of bore-bourrelet, barrel whip.

5. Transitional ballistic parameters, e.g., muzzle blast, aerodynamic jump, gas flow around existing projectile, initial yaw angle, maximum yaw angle.

Statistical variations in all of the above factors and in the muzzle velocity, which is the area of present interest, will cause variations in the trajectory and hence variations in the ballistic error on target. The total ballistic error can then be given as

$$\sigma_r = (\sigma_v^2 + \sigma_m^2 + \sigma_d^2 + \sigma_j^2 + \sigma_a^2 + \ldots)^{\frac{1}{2}}$$

where the subscripts v, m, d, j, a refer to the "independently"

assumed sources of, respectively, muzzle velocity, projectile mass, drag coefficient, jump, yaw angle, etc. Denoting by the quantity σ_e, the component of ballistic linear standard deviation due to all effects other than velocity variation, then we obtain,

$$\sigma_r = (\sigma_v^2 + \sigma_e^2)^{1/2} \tag{2.1}$$

as the fundamental relation used in this study for ballistic linear standard deviation.

FORMULATION

The total delivery error of a system with only aiming error and ballistic error is explicitly,

$$\sigma_t = (\sigma_a^2 + \sigma_r^2)^{1/2} \tag{3.1}$$

The hit probability on a rectangle with horizontal and vertical sides of $2a_x$ and $2a_y$, respectively, is given by the following expression[7] containing the Gaussian (normal) cumulative distribution function $\alpha(t)$.

$$P_h = \tfrac{1}{4}\left[\alpha\left(\frac{a_x - \mu_x}{\sigma_{tx}}\right) + \alpha\left(\frac{a_x + \mu_x}{\sigma_{tx}}\right)\right] \left[\alpha\left(\frac{a_y - \mu_y}{\sigma_{ty}}\right) + \alpha\left(\frac{a_y + \mu_y}{\sigma_{ty}}\right)\right] \tag{3.2}$$

where the subscripts x and y are the horizontal deflection coordinate and vertical coordinate, where μ_x and μ_y are the biases (means) of these orthogonal distributions, and where

$$\alpha(t) = \int_{-t}^{t} \Psi(\tau)d\tau = 2\int_{-\infty}^{t} \Psi(\tau)d\tau - 1 \tag{3.3}$$

$$\Psi(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2} \tag{3.4}$$

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[7] Malinoski, F.A., A Summary of Mathematical Methods in Hit and Incapacitation Probability Analysis of Small Arms Weapons Systems (U), Frankford Arsenal, Report R-1831, Dec. 1966. (Confidential)

The function $\Psi(t)$ is the normal density function. Equation (3.2) is the rectangular heteroscedastic* case with bias for the generalized Gaussian bivariate distribution, in which the two-dimensional distribution has degenerated into a product of two independent Gaussian univariates, because of the simplicity of the target geometry. Tables of $\alpha(t)$ are widely available.[8 to 10]

The calculation of data on $\bar{V}$ and $\bar{\sigma}_V$ in terms of errors on target is accomplished by considering the trajectories of the projectile corresponding to three velocities $\bar{V} + \bar{\sigma}_V$, $\bar{V}$, and $\bar{V} - \bar{\sigma}_V$. These trajectories are computed by considering the standard drag function for the given projectile and by assuming a constant reasonable value of crosswind and a constant angle of fire. As functions of the range z, the linear standard deviations on target (in distance units or angular units of mils) due to the velocity variation are,

$$\sigma_{vx}(z) = \tfrac{1}{2}[x_+(z) - x_-(z)] \tag{3.5a}$$

$$\sigma_{vy}(z) = \tfrac{1}{2}[y_+(z) - y_-(z)] \tag{3.5b}$$

where the subscripts + and - refer to trajectory values for $\bar{V} + \bar{\sigma}_V$ and $\bar{V} - \bar{\sigma}_V$. When $\bar{\sigma}_V \ll \bar{V}$, Equations (3.5) become,

$$\sigma_{vx}(z) \simeq x_+(z) - \bar{x}(z) \simeq \bar{x}(z) - x_-(z) \tag{3.6a}$$

$$\sigma_{vy}(z) \simeq y_+(z) - \bar{y}(z) \simeq \bar{y}(z) - y_-(z) \tag{3.6b}$$

where $\bar{x}(z)$ and $\bar{y}(z)$ are trajectory coordinates corresponding to the average velocity $\bar{V}$.

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[8] Burington, R.S., and May, D.C., Jr., Handbook of Probability and Statistics with Tables, Handbook Publishers, Inc., Sandusky, Ohio, 1953.

[9] Burington, R.S., Handbook of Mathematical Tables and Formulas, McGraw-Hill, New York, 1965.

[10] Tables of Normal Probability Functions, National Bureau of Standards, Applied Mathematics Series, No. 23, 1953.

* Having unequal horizontal and vertical variances. (Homoscedastic refers to a distribution with equal variances in these directions.)

Denoting the bias values of the base tap and nose tap conditions of the air space by b and n, respectively, we obtain,

$$\mu_{nx}(z) = \mu_{bx}(z) + \bar{x}_n(z) - \bar{x}_b(z) \tag{3.7a}$$

$$\mu_{ny}(z) = \mu_{by}(z) + \bar{y}_n(z) - \bar{y}_b(z) \tag{3.7b}$$

In the case where the sight adjustments of the weapon can cause the biases of, say, the base tap round to vanish at each range, Equations (3.7) become,

$$\mu_{nx}(z) = \bar{x}_n(z) - \bar{x}_b(z) \tag{3.8a}$$

$$\mu_{ny}(z) = \bar{y}_n(z) - \bar{y}_b(z) \tag{3.8b}$$

ASSUMPTIONS, PROCEDURE, AND DISCUSSION

Since the principal objective of this study is to investigate the effect of velocity-air space variation upon system performance, certain conditions are assumed which tend to maximize the small influence of this velocity variation. One of these is the implicit assumption in Equation (2.1) of no range estimation error for the sight settings of the weapon. For small arms fire with no optical or laser range finders, the range estimation error is approximately 20 percent of the actual range. The corresponding vertical error on target is at least one order of magnitude larger than the error due to velocity variation σ_{vy} but is neglected in order not to obscure the influence of velocity variation.

The second assumption is that σ_e (in mils) in Equation (2.1) remains constant with range, although this quantity probably does increase slightly with range in an undetermined manner. These two conditions constitute a statement that the dependence of σ_r with range is due to only the velocity variation term σ_v.

A third assumption is that no error would be introduced by the geometry of the sight setting itself, considered to be sufficiently finely divided so that for both height and windage, zero bias ($\mu_x = 0$, $\mu_y = 0$) at all ranges is admissable for the standard, base tap (BT) data. The net effect of the exclusion of these three additional sources of error is to enhance the effect of changes in $\bar{V}$ and $\bar{\sigma}_v$ upon the hit probability to the greatest extent possible.

The BT data is essentially considered as the reference condition around which the small perturbations of $\bar{V}$ and $\bar{\sigma}_v$ of the nose tap (NT) data can be examined.

The most crucial assumption in this study is that of the aiming error, since this is the most influential parameter in hit probability computations. Two levels of σ_A are chosen, as a consequence of their explicit and implicit definition in various current requirements documents (e.g., Materiel Need, Letter of Agreement) for the Future Rifle System (FRS). The explicit value of σ_A is one mil, which is associated with a long range hit probability requirement. This value is quite low for a combat aiming error but is likely achievable in a small percentage of firers under idealized conditions of very small time stress (i.e., relatively large time to fire) and counter-fire stress. The second assumption of aiming error is the day defense case, which represents larger, more realistic values of σ_A, a quasi-tactical estimate in a defensive role with greater time and counter-fire stresses. The day defense aiming errors σ_A (in mils) are given by the following equation as a function of impulse and range.[5,11,12]

$$\sigma_A = \left[\left(\frac{a_0 J + a_1}{z + a_2} + a_3\right)^2 - a_4^2\right]^{1/2} \qquad (4.1)$$

where J = muzzle impulse, z = range, and the a's are constants. The constant a_4 (0.4 mils) is necessary to eliminate the ballistic error from the experimentally determined delivery error (first term in parenthesis).

In Table II, the three trajectories to a range of 500 meters for each distinct M80 system are presented, corresponding to the three velocities $\bar{V} + \bar{\sigma}_v$, $\bar{V}$, and $\bar{V} - \bar{\sigma}_v$. A crosswind of 10mph is assumed for the positive values of deflection x. A zero angle of fire applies in all trajectory calculations.

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[5] Simmons, R.L., and Cain, R.E., Effectiveness of Small Arms Weapons Systems (U), Ballistic Research Laboratories, Memorandum Report 1764, July 1966. (Confidential)

[11] Fallin, H., Evaluation of AAI SFR, Letter to CO, USASASA, 10 Nov 1969. (Confidential)

[12] Malinoski, F.A., Small Arms Systems Analysis, Munitions Command Infantry Ammunition Seminar, May, 1971. (Confidential)

TABLE II.

The BT and NT M80 Trajectories of Velocity, Deflection x, and Ordinate y vs. Range for Three Velocity Levels, $\overline{V} + \overline{\sigma}_v$, $\overline{V}$, and $\overline{V} - \overline{\sigma}_v$, with a Zero Angle of Fire and a Crosswind of 10 mph.

Velocities in fps
Coordinates x and y in inches

M80 Cartridge Description	Range z (Meters)	V_+	x_+	y_+	$\overline{V}$	$\overline{x}$	$\overline{y}$	V_-	x_-	y_-
LC, BT ($\overline{V}$ = 2745,	100	2576	.9618	- 2.7768	2558	.9708	- 2.8134	2541	.9800	- 2.8507
$\overline{\sigma}_v$ = 18	200	2341	4.0270	-11.8475	2324	4.0659	-12.0068	2306	4.1054	-12.1693
at 78 feet)	300	2118	9.5090	-28.5421	2103	9.6027	-28.9326	2087	9.6976	-29.3296
	400	1907	17.7913	-54.5525	1892	17.9701	-55.3111	1678	18.1521	-56.0856
	500	1707	29.3468	-92.0578	1693	29.6497	-93.3672	1679	29.9578	-94.7031
LC, NT ($\overline{V}$ = 2767,	100	2601	.9484	- 2.7230	2579	.9598	- 2.7688	2557	.9713	- 2.8155
$\overline{\sigma}_v$ = 23 at	200	2365	3.9762	-11.6155	2344	4.1085	-11.8128	2323	4.0680	-12.0157
78 feet)	300	2141	9.3722	-27.9735	2121	9.4885	-28.4567	2102	9.6079	-28.9544
	400	1929	17.5285	-53.4422	1910	17.7520	-54.3862	1892	17.9801	-55.3538
	500	1728	28.9830	-90.1469	1710	29.2801	-91.7704	1692	29.6666	-93.4404
TC, BT ($\overline{V}$ = 2748,	100	2576	.9612	- 2.7748	2561	.9693	- 2.8073	2545	.9775	- 2.8403
$\overline{\sigma}_v$ = 16 at	200	2342	4.0249	-11.8388	2327	4.0593	-11.9800	2312	4.0944	-12.1239
78 feet)	300	2119	9.5038	-28.5206	2105	9.5870	-28.8671	2091	9.6711	-29.2187
	400	1908	17.7815	-54.5109	1895	17.9400	-55.1833	1882	18.1013	-55.8693
	500	1708	29.3300	-91.9858	1695	29.5990	-93.1475	1683	29.8714	-94.3284
TC, NT ($\overline{V}$ = 2773,	100	2605	.9464	- 2.7152	2585	.9568	- 2.7568	2565	.9673	- 2.7991
$\overline{\sigma}_v$ = 21 at	200	2369	3.9619	-11.5817	2350	4.0059	-11.7609	2331	4.0506	-11.9444
78 feet)	300	2145	9.3521	-27.8905	2127	9.4580	-28.3297	2109	9.5662	-28.7801
	400	1932	17.4903	-53.2812	1915	17.6934	-54.1381	1898	17.9000	-55.0136
	500	1731	28.8381	-89.8679	1715	29.1812	-91.3439	1699	29.5315	-92.8559

Table III shows similar data for the M193 systems. The coordinates are converted to units of mils by the following equation:

$$x(\text{mils}) = 1018.6\ x/z \tag{4.2}$$

where x = target coordinate in arbitrary units of length, and z = range to target in the same units as x. The horizontal and vertical values of $\sigma_v(z)$, $\sigma_e(z)$, $\sigma_r(z)$ and $\mu(z)$ are given in Tables IV and V for the M80 and M193 cartridges. First the $\sigma_v(z)$ values are calculated by Equations (3.5) and the trajectory values in Tables II and III. The values of $\sigma_e(z)$ are computed by Equation (2.1) with the assumption that at 100 meters, $\sigma_r(z)$ is precisely 0.4 mils. This value of $\sigma_e(z)$, calculated in mils, is then considered to remain constant at all ranges. The biases of the BT rounds are assumed to be zero, and the $\mu_n(z)$ values (those of the NT rounds) were calculated by Equations (3.8). The degree of precision displayed in Tables II to V is a necessary artifice to allow the evaluation of the velocity variation effect, although it is recognized that the inherent precision in quantities such as σ_e and $\sigma_r(z)$ is considerably less than shown. The largest variation in the mil values of σ's and μ's in Tables IV and V occurs at 500 meters, as expected, due to the increasing curvature of the trajectories. However, the variations due to the NT procedure, in σ_{rx}, μ_x, σ_{ry}, and μ_y are quite small. The introduction of the finite bias by the NT procedure is probably more important than the slight increase of σ_r.

As mentioned previously, one case of assumed aiming error $\sigma_A(z)$ is given by Equation (4.1), in which the delivery error $\sigma_D(z)$ for the day defense posture was

$$\sigma_D(z) = \frac{a_0 J + a_1}{z + a_2} + a_3 \tag{4.3}$$

Since this expression is used as a base around which the velocity variation terms are applied as perturbations, values of $\sigma_A(z)$ and $\sigma_D(z)$ in mils are calculated for the M80 and M193, for which the recoil momenta, J, are 2.60 and 1.23 lb-sec, respectively. These values for the day defense case are not shown because of the security classification.* However, it can be stated that the values of $\sigma_D(z)$ and $\sigma_A(z)$ are considerably larger than the $\sigma_r(z)$ values in Tables IV and V and therefore dominate the total delivery errors of Equation (3.1).

* These data are available to qualified requestors from the authors.

TABLE III.

The BT and NT M193 Trajectories of Velocity, Deflection x and Ordinate y vs. Range for Three Velocity Levels, $\bar{V} + \bar{\sigma}_V$, $\bar{V}$, and $\bar{V} - \bar{\sigma}_V$, Zero Angle of Fire and a Crosswind of 10 mph.

Velocities in fps
Coordinates x and y in inches

M193 Cartridge Description	Range R (Meters)	V_+	x_+	y_+	$\bar{V}$	$\bar{x}$	$\bar{y}$	V_-	x_-	y_-
LC, BT	100	2856	1.2739	- 2.1186	2839	1.2843	- 2.1424	2823	1.2950	- 2.1669
($\bar{V}$ = 3246,	200	2461	5.4764	- 9.3808	2446	5.5225	- 9.4897	2430	5.5691	- 9.6005
$\bar{\sigma}_V$ = 18	300	2098	13.3116	-23.5791	2084	13.4263	-23.8605	2070	13.5428	-24.1477
at 78 feet)	400	1765	25.7410	-47.3368	1753	25.9703	-47.9204	1740	26.2040	-48.5174
	500	1459	44.1443	-84.6326	1447	44.5579	-85.7215	1435	44.9796	-86.8351
LC, NT	100	2883	1.2571	- 2.0807	2859	1.2716	- 2.1133	2837	1.2861	- 2.1465
($\bar{V}$ = 3268,	200	2486	5.4035	- 9.2089	2464	5.4663	- 9.3568	2443	5.5302	- 9.5080
$\bar{\sigma}_V$ = 25	300	2121	13.1296	-23.1345	2101	13.2863	-23.5171	2082	13.4455	-23.9078
at 78 feet)	400	1786	25.3767	-46.4136	1768	25.6904	-47.2082	1750	26.0090	-48.0190
	500	1478	43.4684	-82.9122	1461	44.0531	-84.3928	1445	44.6276	-85.9054
TC, BT	100	2863	1.2698	- 2.1093	2843	1.2820	- 2.1371	2824	1.2945	- 2.1656
($\bar{V}$ = 3250,	200	2467	5.4585	- 9.3385	2449	5.5122	- 9.4654	2431	5.5666	- 9.5946
$\bar{\sigma}_V$ = 21	300	2104	13.2671	-23.4700	2087	13.4007	-23.7977	2071	13.5366	-24.1324
at 78 feet)	400	1771	25.6517	-47.1099	1755	25.9189	-47.7895	1740	26.1917	-48.4858
	500	1463	43.9832	-84.2095	1449	44.4651	-85.4769	1435	44.9572	-86.7758
TC, NT	100	2859	1.2721	- 2.1146	2838	1.2855	- 2.1415	2816	1.2992	- 2.1766
($\bar{V}$ = 3244,	200	2464	5.4687	- 9.3624	2444	5.5276	- 9.5019	2424	5.5874	- 9.6443
$\bar{\sigma}_V$ = 23	300	2101	13.2922	-23.5316	2083	13.4391	-23.8929	2065	13.5886	-24.2613
at 78 feet)	400	1768	25.7022	-47.2382	1751	25.9961	-47.9861	1735	26.2962	-48.7533
	500	1461	44.0744	-84.4488	1445	44.6044	-85.8400	1430	45.1765	-87.2766

TABLE IV.

The Components of Linear Standard Deviation σ_e, $\sigma_v(\mathfrak{s})$, and $\sigma_r(\mathfrak{s})$ and Bias $\mu(\mathfrak{s})$ vs. Range $\mathfrak{s}$ for the BT and NT M80 Cartridges

All σ's and μ's in mils.

M80 Cartridge Description	Range (Meters)	σ_{ey}	$\sigma_{vy}(\mathfrak{s})$	$\sigma_{ry}(\mathfrak{s})$	$\mu_y(\mathfrak{s})$	σ_{ex}	$\sigma_{vx}(\mathfrak{s})$	$\sigma_{rx}(\mathfrak{s})$	$\mu_x(\mathfrak{s})$
LC, BT	100	.39989	.00955	.40000	0	.39999	.00235	.40000	0
	200	.39989	.02081	.40043	0	.39999	.00507	.40002	0
	300	.39989	.03396	.40132	0	.39999	.00813	.40008	0
	400	.39989	.05022	.40303	0	.39999	.01167	.40016	0
	500	.39989	.06844	.40570	0	.39999	.01581	.40030	0
LC, NT	100	.39982	.01195	.40000	.011556	.39989	.00298	.40000	.002867
	200	.39982	.02589	.40060	.025096	.39989	.00633	.40003	.006124
	300	.39982	.04230	.40205	.041037	.39989	.01017	.40012	.009848
	400	.39982	.06182	.40457	.059826	.39989	.01461	.40026	.014105
	500	.39982	.08521	.40880	.082625	.39989	.01976	.40048	.019123
TC, BT	100	.39991	.00847	.40000	0	.39999	.00210	.40000	0
	200	.39991	.01844	.40034	0	.39999	.00450	.40002	0
	300	.39991	.03010	.40104	0	.39999	.00721	.40006	0
	400	.39991	.04393	.40232	0	.39999	.01034	.40013	0
	500	.39991	.06061	.40448	0	.39999	.01401	.40024	0
TC, NT	100	.39985	.01086	.40000	.013075	.39999	.00271	.40000	.003247
	200	.39985	.02346	.40054	.028334	.39999	.005[illegible]4	.40003	.006917
	300	.39985	.03836	.40169	.046340	.39999	.00[illegible]23	.40010	.011216
	400	.39985	.05603	.40376	.067603	.39999	.01322	.40021	.015951
	500	.39985	.07731	.40726	.093331	.39999	.01794	.40039	.021614

TABLE V.

The Components of Linear Standard Deviations σ_e, $\sigma_v(•)$, and $\sigma_r(•)$ and Bias $\mu(•)$ vs. Range for the BT and NT M193 Cartridges

All σ's and μ's in mils.

M193 Cartridge Description	Range (Meters)	σ_{ey}	$\sigma_{vy}(•)$	$\sigma_{ry}(•)$	$\mu_y(•)$	σ_{ex}	$\sigma_{vx}(•)$	$\sigma_{rx}(•)$	$\mu_x(•)$
LC, BT	100	.39951	.00625	.40000	0	.39999	.00273	.40000	0
	200	.39951	.01421	.40020	0	.39999	.00600	.40004	0
	300	.39951	.02452	.40070	0	.39999	.00997	.40012	0
	400	.39951	.03818	.40177	0	.39999	.01497	.40027	0
	500	39951	.05699	.40399	0	.39999	.02161	.40057	0
LC, NT	100	.39991	.00851	.40000	.007357	.39998	.00375	.40000	.003286
	200	.39991	.01934	.40038	.017142	.39998	.00820	.40007	.007270
	300	.39991	.03334	.40130	.029621	.39998	.01362	.40021	.012074
	400	.39991	.05192	.40327	.046068	.39998	.02044	.40050	.018104
	500	.39991	.07744	.40734	.068753	.39998	.02948	.40106	.026121
TC, BT	100	.39993	.00728	.40000	0	.39999	.00320	.40000	0
	200	.39993	.01657	.40028	0	.39999	.00699	.40005	0
	300	.39993	.02856	.40095	0	.39999	.01162	.40016	0
	400	.39993	.04450	.40240	0	.39999	.01746	.40037	0
	500	.39993	.06640	.40541	0	.39999	.02520	.40078	0
TC, NT	100	.39992	.00802	.40000	-.001141	.39998	.00351	.40000	-.000906
	200	.39992	.01823	.40034	-.004762	.39998	.00768	.40006	-.001992
	300	.39992	.03147	.40116	-.008137	.39998	.01279	.40019	-.003312
	400	.39992	.04900	.40291	-.012721	.39998	.01921	.40045	-.004993
	500	.39992	.07316	.40656	-.018997	.39998	.02774	.40094	-.007208

Included in Table VI are the target dimensions a_x and a_y in mils. The target considered is the type E target (width = $2a_x$ = 0.4953 meters; height = $2a_y$ = 0.8611 meters).

TABLE VI.

Angular Dimensions of the Type E Target vs. Range

Half-width = a_x = .24765 meters
Half-height = a_y = .43055 meters

Range (meters)	$a_x(z)$ (mils)	$a_y(z)$ (mils)
100	2.522563	4.385582
200	1.261282	2.192791
300	.840854	1.461861
400	.630641	1.096396
500	.504513	.877116

Since the total delivery error distributions are considered to be heteroscedastic, the x and y components of $\sigma_t(z)$ and the $P_h(z)$ values for the M80 and M193 are shown in Tables VII and VIII, for the one mil aiming error case. For the day defense case (also not shown here), the values of $\sigma_t(z)$ and $P_h(z)$ are, respectively, considerably larger than and smaller than the corresponding results of the one mil aiming error case. The quantity $\sigma_t(z)$ is calculated by Equation (3.1) which for a constant σ_e(100 meters) value and a one mil aiming error, becomes (in mils),

$$\sigma_t(z) = [1^2 + \sigma_r^2(z)]^{\frac{1}{2}}$$

$$\sigma_t(z) = [1^2 + \sigma_e^2 + \sigma_v^2(z)]^{\frac{1}{2}}$$

$$\sigma_t(z) = [1 + (0.4)^2 + \sigma_v^2(z) - \sigma_v^2(100 \text{ meters})]^{\frac{1}{2}} \qquad (4.4)$$

TABLE VII.

The BT and NT M80 Total Delivery Errors $\sigma_t(z)$ and Hit Probability $P_h(z)$ on a Type E Target vs. Range z with a One Mil Aiming Error.

All σ's in mils.

M80 Cartridge Description	Range z (Meters)	$\sigma_{ty}(z)$	$\sigma_{tx}(z)$	$P_h(z)$
LC,BT	100	1.07703	1.07703	.98089
	200	1.07719	1.07704	.72670
	300	1.07753	1.07706	.46590
	400	1.07812	1.07709	.30516
	500	1.07916	1.07715	.21035
LC,NT	100	1.07703	1.07703	.98089
	200	1.07728	1.07705	.72669
	300	1.07780	1.07708	.46602
	400	1.07874	1.07713	.30535
	500	1.08033	1.07721	.20979
TC,BT	100	1.07703	1.07703	.98089
	200	1.07716	1.07704	.72666
	300	1.07742	1.07706	.46623
	400	1.07790	1.07708	.30570
	500	1.07876	1.07712	.21047
TC,NT	100	1.07703	1.07703	.98089
	200	1.07723	1.07704	.72665
	300	1.07766	1.07707	.46592
	400	1.07844	1.07711	.30465
	500	1.07975	1.07717	.21045

TABLE VIII.

The BT and NT M193 Total Delivery Errors $\sigma_t(R)$ and Hit Probability $P_h(R)$ on a Type E Target vs. Range R with a One Mil Aiming Error.

All σ's in mils

M193 Cartridge Description	Range R (Meters)	$\sigma_{ty}(R)$	$\sigma_{tx}(R)$	$P_h(R)$
LC, BT	100	1.07703	1.07703	.98089
	200	1.07711	1.07705	.72671
	300	1.07729	1.07708	.46625
	400	1.07769	1.07713	.30523
	500	1.07852	1.07725	.21044
LC, NT	100	1.07703	1.07703	.98088
	200	1.07717	1.07706	.72670
	300	1.07752	1.07711	.46605
	400	1.07825	1.07722	.30485
	500	1.07973	1.07743	.20983
TC, BT	100	1.07703	1.07703	.98089
	200	1.07714	1.07705	.72670
	300	1.07739	1.07709	.46604
	400	1.07793	1.07717	.30517
	500	1.07905	1.07732	.21034
TC, NT	100	1.07703	1.07703	.98089
	200	1.07716	1.07706	.72670
	300	1.07746	1.07710	.46602
	400	1.07812	1.07720	.30512
	500	1.07948	1.07738	.21022

For the day defense case, the total delivery error, by Equation (3.1) is,

$$\sigma_t(z) = [\sigma_D^2(z) - (0.4)^2 + \sigma_e^2 + \sigma_v^2(z)]^{\frac{1}{2}}$$

$$\sigma_t(z) = [\sigma_D^2(z) - (0.4)^2 + (0.4)^2 - \sigma_v^2(100 \text{ meters}) + \sigma_v^2(z)]^{\frac{1}{2}}$$

$$\sigma_t(z) = [\sigma_D^2(z) + \sigma_v^2(z) - \sigma_v^2(100 \text{ meters})]^{\frac{1}{2}} \qquad (4.5)$$

Equations (4.4) and (4.5) show the explicit dependence upon the velocity variation term. The velocity variation distribution is considered to be heteroscedastic in these two equations, reflecting the horizontal and vertical trajectories, whereas, the component of the day defense total error distribution due to the day defense equation, i.e., Equation (4.3), is assumed to be homoscedastic. The P_h of the rectangular type "E" target is then computed by Equation (3.2). The changes in the $\sigma_t(z)$ and $P_h(z)$ values due to the NT procedure are quite small for the one mil aiming error case and are still smaller for the day defense errors.

The percentage change in $P_h(z)$, caused by the NT method, is presented in Table IX, for both cartridges and for both conditions of aiming error. It is seen that the effect of the velocity variations caused by the NT method is very insignificant. The differences shown for the day defense case, i.e., -0.02 percent, are near the limits of precision of the numerical methods for hit probability.

In Table X, the limiting case of vanishing aiming error is considered for the M80, LC and M193, LC cartridges, at a range of 500 meters, which shows the maximum change in hit probability. It is seen that the change in P_h due to the NT method is also quite small, only -0.6 percent, even for this extreme case of no aiming error, which allows the velocity variations to be unrealistically dominant to the greatest possible extent.

TABLE IX.

Maximum Percentage Changes in the Hit Probability of the NT M80 and M193 Cartridges, Relative to the BT Cartridges, for the Two Levels of Aiming Error σ_A.

$\Delta P_h = P_{hn} - P_{hb}$

Percent = 100 Max$\Delta P_h / P_{hb}$

Type E Target

Cartridge Description	σ_A	Max. ΔP_h	Percent
M80, LC	one mil	-.00057	-0.3
M80, TC	one mil	-.00055	-0.3
M193, LC	one mil	-.00060	-0.3
M193, TC	one mil	-.00012	-0.1
M80, LC	Day Defense	-.00002	-0.04
M80, TC	Day Defense	-.00002	-0.04
M193, LC	Day Defense	-.00001	-0.02
M193, TC	Day Defense	-.00001	-0.02

TABLE X.

For limiting Case of Zero Aiming Error, the Maximum Change in Hit Probability at 500 Meters, Due to the NT Method for M80, LC and M193, LC Cartridges.

$\sigma_t = \sigma_r$

$\Delta P_h = P_{hn} - P_{hb}$

Percent = 100 $\Delta P_h / P_{hb}$

Type E Target

Cartridge Description	P_h (500 meters)		ΔP_h	Percent
	BT	NT		
M80, LC	.76824	.76370	-.00454	-0.6
M193, LC	.76855	.76391	-.00464	-0.6

OTHER TERMINAL EFFECTS OF VELOCITY VARIATIONS

Another possible influence of velocity variations of projectiles is in the area of down-range conditional probabilities of causing target effects, e.g., quantities such as P(I/H), the probability of incapacitation, given a hit and P(p/H), the probability of penetrating some target material of assumed thickness, when given a hit. These conditional probabilities are not treated in this study but were recognized as additional affected parameters, independent of hit probability. Statistical data on the variances of these conditional probabilities are not generally very complete, but where such data are available, it is apparent that the inherent experimental linear standard deviations of these conditional probabilities are much greater than shifts due to a 1 percent change in reference velocity. The principal reasons for the large stochastic noise in the conditional probability data are the high degree of variability in the initial impact conditions and in the complexity of the dynamical processes of penetration phenomena in various media. Therefore, the neglect of small velocity-dependent variances in these conditional probabilities is justifiable within the framework of the existing experimental methodologies.

CONCLUDING REMARKS

The principal result of these computations is that the effect on hit probability of the location of airspace in the standard cartridges, M80 and M193, is far from critical; it is virtually insignificant. The largest changes resulting from the NT procedure in the normal bivariate hit probabilities on a Type E target occur at the maximum range considered, i.e., 500 meters, and are -0.04 percent for day defense aiming errors, -0.3 percent for a one mil aiming error, and -0.6 percent for the limiting case of zero aiming error. These insignificant changes are obtained even though several assumptions are made which permit the maximum possible dominance of the velocity-airspace variations, namely, the neglect of range estimation error and sight setting error and the treatment of σ_e as range-invariant. The relative unimportance of airspace position for these two cartridges is due to the fact that the NT procedure produces a change of less than 1 percent in $\overline{V}$ and less than 40 percent in $\overline{\sigma}_V$. Although it is true that other systems having greater variations in $\overline{V}$ and $\overline{\sigma}_V$ might show larger effects upon hit probability, it is highly unlikely that any small arms system would have velocity-dependent shifts in errors and biases sufficiently large to have a significant influence on hit probability, even for very small aiming errors.

REFERENCES

1. Moore, L., Proposal-Engineering Test of Small Caliber Rifle-Ammunition Systems, 17 July, 1974.

2. Carn, R.E., Simmons, R.L., and Sperrazza, J., Comparative Effectiveness Evaluation of the M14 and Other Rifle Concepts (U), Ballistic Research Laboratories, Tech. Note 1482, Dec. 1962. (Secret)

3. Malinoski, F.A., A Casualty Probability Analysis of Small Arms Weapons Systems of Various Calibers (U), Frankford Arsenal, Report R-1712, March 1964. (Secret)

4. Malinoski, F.A., and McHugh, R.J., An Effectiveness Analysis of Spin-Stabilized Rifle Systems Based on a Caliber .17 Projectile (U), Frankford Arsenal, Report R-1804, Feb. 1966. (Confidential)

5. Simmons, R.L., and Carn, R.E., Effectiveness of Small Arms Weapons Systems (U), Ballistic Research Laboratories, Memorandum Report 1764, July 1966. (Confidential)

6. Carn, R.E., and Fallin, H.K., Effectiveness Comparison of 1:12 and 1:14 Inch Twist Rates for M16A1 Rifle (U), Ballistic Research Laboratories, Memo. Report 1886, Dec. 1968. (Secret)

7. Malinoski, F.A., A Summary of Mathematical Methods in Hit and Incapacitation Probability Analysis of Small Arms Weapons Systems (U), Frankford Arsenal, Report R-1831, Dec. 1966. (Confidential)

8. Burington, R.S., and May, D.C., Jr., Handbook of Probability and Statistics with Tables, Handbook Publishers, Inc., Sandusky, Ohio, 1953.

9. Burington, R.S., Handbook of Mathematical Tables and Formulas, McGraw-Hill, New York, 1965.

10. Tables of Normal Probability Functions, National Bureau of Standards, Applied Mathematics Series, No. 23, 1953.

11. Fallin, H., Evaluation of AAI SFR, Letter to CO, USASASA, 10 Nov 1969. (Confidential)

12. Malinoski, F.A., Small Arms Systems Analysis, Munitions Command Infantry Ammunition Seminar, May, 1971. (Confidential)

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